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Portfolio Optimization, CAPM & Factor Modeling Project

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by

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Abstract

In this project, we implement portfolio theory to construct our portfolio, applying the theory to real practice. There are 3 parts in this project, including portfolio optimization, Capital Asset Pricing Model (CAPM) analysis and Factor Model analysis. We implement portfolio theory in the portfolio optimization part. In the second part, we use the CAPM to analyze and improve our portfolio. In the third part we extend our CAPM to factor models to get a deeper analysis of our portfolio.

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INTRODUCTION

Portfolio theory is based upon two principles: maximization of expected return and minimization of risk. These goals are somewhat at odds because riskier assets generally have a higher expected return, since investors demand a reward for bearing risk. The difference between the expected return of a risky asset and the risk-free rate of return is called the risk premium. Without risk premiums, few investors would invest in risky assets.

Thus, we try to find a way to minimize risk and get higher expected return. According to Markowitz portfolio theory, the optimal or efficient portfolio mixes the tangency portfolio with the risk-free asset. Each efficient portfolio has two properties:

- It has a higher expected return than any other portfolio with the same risk and
- It has a smaller risk than any other portfolio with the same expected return.

In this project, we implement portfolio theory to construct our portfolio, applying the theory to real practice.

There are 3 parts in this project, including portfolio optimization, Capital Asset Pricing Model (CAPM) analysis and Factor Model analysis. We implement portfolio theory in the portfolio optimization part. In the second part, we use the CAPM to analyze our portfolio. In the third part we extend our CAPM to factor models to get a deeper analysis of our portfolio.

We implement portfolio theory in the portfolio optimization part. In the second part, we use the CAPM to analyze and improve our portfolio. In the third part we extend our CAPM to factor models to get a deeper analysis of our portfolio.

In the portfolio optimization part, we use our Interactive Brokers (Interactive Brokers, 2011) account, form a long position in the tangency portfolio using $500, 000 as initial capital.

ASSETS

For diversification purposes, we try to choose assets across different industries. The 15 assets we choose include retailer stores, financial companies, energy companies, environmental companies, technology companies, health care and medical companies, real estate, airline, advertising companies and weapons & military including commercial electronics companies, which are all listed in the NYSE. They are listed as follows:

WMT: Wal-Mart Stores, Inc. Common St. WMT. It is an American public multinational corporation, which is the world's 18th largest public corporation, according to the Forbes Global 2000 list (Forbes.com, 2000), and the largest public corporation when ranked by revenue. It is also the biggest private employer in the world with over 2 million employees, and is the largest retailer in the world.

NGG: National Grid plc. It is a multinational electricity and gas utility company headquartered in London, United Kingdom. Its principal activities are in the United Kingdom and northeastern United States and it is one of the largest investor-owned energy companies in the world.

COH: Coach, Inc. It is an upscale American leather goods company known for ladies' and men's handbags, as well as items such as luggage, briefcases, wallets and other accessories.

JPM: JPMorgan Chase & Co. It is an American multinational banking corporation consisting of securities, investments and retail. It is the largest bank in the United States by assets and market capitalization.

CVX: Chevron Corporation. It is an American multinational energy corporation headquartered in San Ramon, California, United States and active in more than 180 countries. It is engaged in every aspect of the oil, gas, and geothermal energy industries, including exploration and production; refining, marketing and transport; chemicals manufacturing and sales; and power generation.

VE: Veolia Environment. It is a multinational French company with activities in four main service and utility areas traditionally managed by public authorities - water supply and water management, waste management, energy and transport services.

WMB: Williams Companies, Inc. It is an energy company based in Tulsa, Oklahoma. Its core business is natural gas exploration, production, processing, and transportation, with additional petroleum and electricity generation assets.

JNJ: Johnson & Johnson. It is an American multinational pharmaceutical, medical devices and consumer packaged goods manufacturer founded in 1886.

DAL: Delta Air Lines, Inc. It is a major airline based in the United States and headquartered in Atlanta, Georgia. The airline operates an extensive domestic and international network serving all continents except Antarctica.

ABT: Abbott Laboratories. It is an American-based global, diversified (multi-division) pharmaceuticals and health care products company.

TXN: Texas Instruments Incorporated. It is an American company based in Dallas, Texas, United States, which develops and commercializes semiconductor and computer technology.

RTN: Raytheon Company. It is a major American defense contractor and industrial corporation with core manufacturing concentrations in weapons and military and commercial electronics. It was previously involved in corporate and special-mission aircraft until early 2007. Raytheon is the world's largest producer of guided missiles.

OMC: Omnicom Group Inc. It is a holding company whose agencies provide marketing and communications services in the disciplines of advertising, customer relationship management strategic media planning and buying, digital and interactive marketing, direct and promotional marketing, public relations and other specialty communications

EQR: Equity Residential. It is a member of the S&P 500, a publicly-traded real estate investment trust based in Chicago, IL.

IBM: International Business Machines Corporation. It is an American multinational technology and consulting corporation headquartered in Armonk, New York, United States. IBM manufactures and sells computer hardware and software, and it offers infrastructure, hosting and consulting services in areas ranging from mainframe computers to nanotechnology.

I. PORTFOLIO OPTIMIZATION

METHOD

According to Markowitz portfolio theory (Ruppert, 2006, p.137-163), we mix risky assets with the risk-free asset, constructing an efficient frontier by using past data. The portfolios that mix the tangency portfolio with the risk-free asset have the maximal sharp ratio, which is a reward-to-risk ratio.

Let $R_{j,t}$ denote the return of asset j, $\mu_{f,t}$ denote the risk-free rate and $\sigma_{j,t}$ denote the risk of asset j, for holding period t. Here we measure risk using standard deviation. The formula for the Sharpe Ratio is

$$\text{Sharpe ratio} = (R_{j,t} - \mu_{f,t})/\sigma_{j,t}$$

We need to find the point in the efficient frontier which maximizes the Sharpe Ratio and this point is the tangency portfolio, which is the optimal portfolio.

PORTFOLIO FORMATION

We use the 15 assets in our portfolio, use the one year Treasury bill rate as our risk-free rate and use past 6 months' data[1] to construct a portfolio to be held for holding period of one week.

We form our portfolio on November 4th, then after the Friday market close we add the new week's data to our former data, compute the new tangency portfolio, and rebalance it on Monday. We close our position on December 5th.

The time length we choose is the last 6 months. We use weekly return to construct our efficient frontier and form our optimal portfolio. As for the risk-free rate, we choose the one year T-bill rate. To convert to a daily rate, we divide it by 52. We form our portfolio on November 4th. The objective here is that we want to see after each Friday the change of our optimal portfolio.

PORTFOLIO REBALANCING

After every Friday market close, we update our data set with the recent week's data, repeat the optimization, and then we get the new optimal portfolio weights. The process is listed as below.

[1] Begin from May, 1st 2011

TABLE 1.1: Portfolio rebalancing on 11/04/2011

	Trade Price	Size	Tangency Portfolio weight
WMT	57.27	1625	0.1862
NGG	50.35	-4173	-0.4202
COH	64.52	-1153	-0.1488
JPM	34.2	-5457	-0.3732
CVX	107.88	1841	0.3973
VE	13.72	-15250	-0.4184
WMB	31.14	7051	0.4392
JNJ	64.06	1161	0.1488
DAL	8.32	-954	-0.0159
ABT	53.49	2234	0.239
TXN	31.53	-363	-0.0229
RTN	44.27	4116	0.3644
OMC	44.03	-9236	-0.8133
EQR	58.51	2618	0.3063
IBM	186.05	3041	1.1317

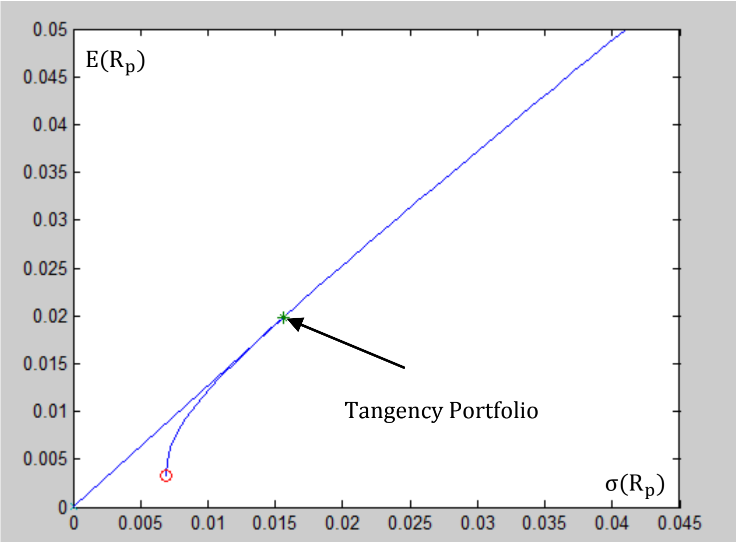

FIGURE 1.1: Tangency Portfolio on 11/04/2011

TABLE 1.2: Portfolio rebalancing on 11/14/2011

	Trade Price	Size	Change of size	Tangency Portfolio Weight
WMT	57.04	1072	-553	0.1223
NGG	50.71	-468	3705	-0.0475
COH	60.66	-852	301	-0.1034
JPM	30.49	-8698	-3242	-0.5304
CVX	99.41	4144	2302	0.8238
VE	11.88	-16258	-1008	-0.3863
WMB	30.43	6734	-317	0.4099
JNJ	63.75	2034	873	0.2594
DAL	7.31	2449	3403	0.0358
ABT	53.74	-156	-2390	-0.0167
TXN	30.27	-2105	-1742	-0.1274
RTN	44.18	-1395	-5510	-0.1232
OMC	41.62	-7486	1749	-0.6232
EQR	54.08	1959	-658	0.2119
IBM	185.19	2957	-85	1.0951

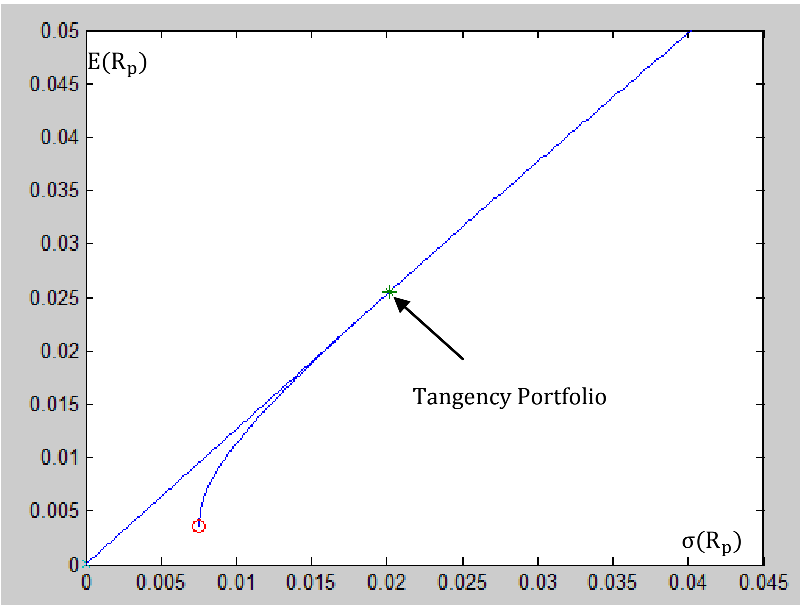

FIGURE 1.2: Tangency Portfolio on 11/14/2011

TABLE 1.3: Portfolio rebalancing on 11/21/2011

	Trade Price	Size	Change of size	Tangency Portfolio Weight
WMT	57.07	1201	128	0.137
NGG	49.7	908	1377	0.0903
COH	50.74	-908	-56	-0.0922
JPM	29.89	-6717	1981	-0.4016
CVX	95.65	2329	-1815	0.4455
VE	11.48	-14365	1892	-0.3298
WMB	29.57	7177	442	0.4244
JNJ	63.07	3095	1061	0.3904
DAL	7.17	2673	224	0.0383
ABT	53.1	104	260	0.011
TXN	29.45	-1956	149	-0.1152
RTN	43.33	-1509	-114	-0.1308
OMC	40.89	-6148	1338	-0.5028
EQR	53.93	1126	-834	0.1214
IBM	182.29	2507	-450	0.9139

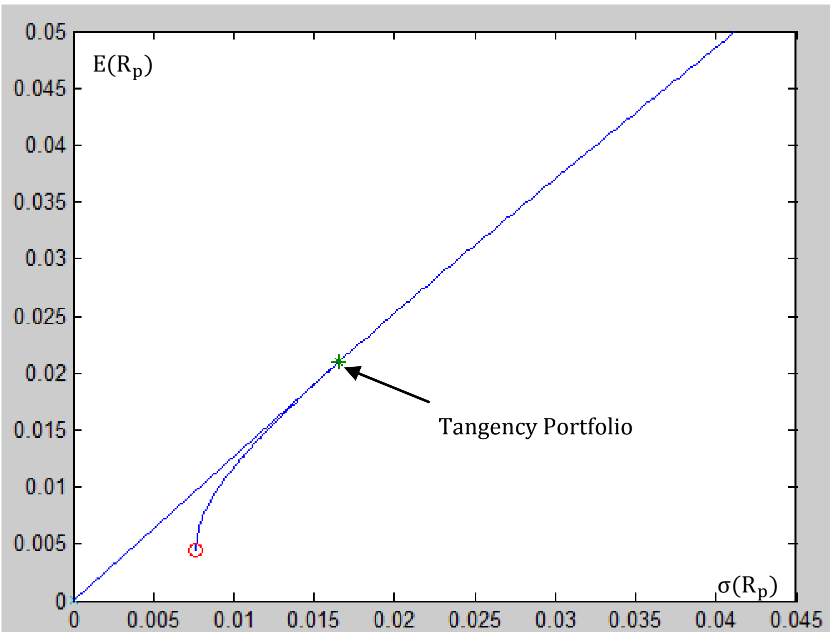

FIGURE 1.3: Tangency Portfolio on 11/21/2011

TABLE 1.4: Portfolio rebalancing on 11/28/2011

	Trade Price	Size	Change of size	Tangency Portfolio Weight
WMT	57.61	5167	3966	0.5953
NGG	49.73	-1100	-2008	-0.1094
COH	61.52	-170	738	-0.0209
JPM	29.96	-13875	-7158	-0.8314
CVX	96.14	3617	1289	0.6955
VE	11.79	-19916	-5551	-0.4696
WMB	30.89	10554	3377	0.652
JNJ	62.53	1775	-1321	0.2219
DAL	7.35	1426	-1247	0.021
ABT	52.89	-2576	-2680	-0.2724
TXN	28.89	-8756	-6800	-0.5059
RTN	44.22	-1208	301	-0.1068
OMC	41.57	-5357	791	-0.4454
EQR	53.6	1557	431	0.1669
IBM	182.38	3863	1357	1.4092

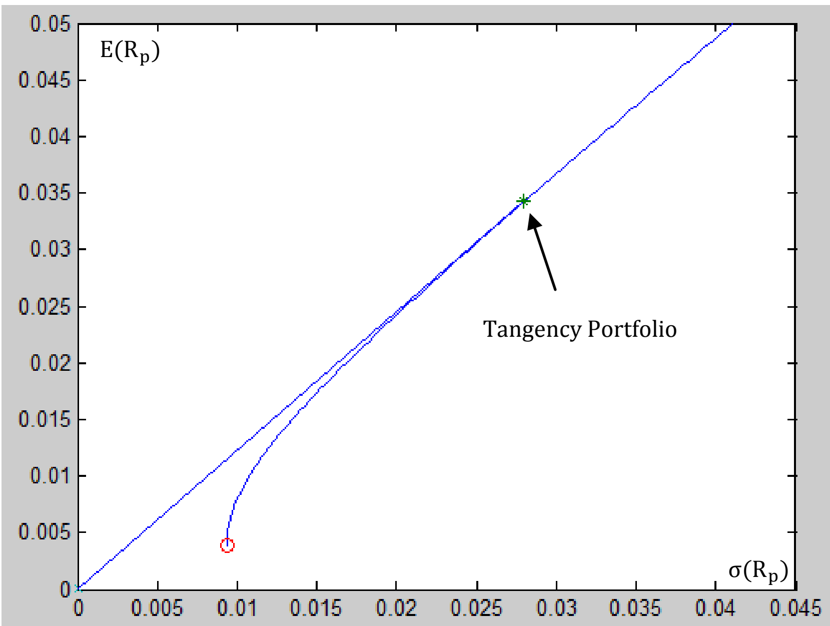

FIGURE 1.4: Tangency Portfolio on 11/21/2011

For the last week, we close our position and get the trade price.

TABLE 1.5: Portfolio closed on 12/5/2011

	Trade Price
WMT	58.26
NGG	49.7
COH	62.87
JPM	33.9
CVX	103.64
VE	12.85
WMB	29.57
JNJ	64
DAL	8.66
ABT	53.1
TXN	30.26
RTN	45.85
OMC	44.49
EQR	54.65
IBM	191.5

Finally, the portfolio performance we get from the report management[2] is as follows:

Realized & Unrealized Performance Summary in Base

	Realized					Unrealized					
Symbol	S/T Profit	S/T Loss	L/T Profit	L/T Loss	Total	S/T Profit	S/T Loss	L/T Profit	L/T Loss	Total	Total
Stocks											
ABT	589.38	-2,908.46	0.00	0.00	-2,319.08	0.00	0.00	0.00	0.00	0.00	-2,319.08
COH	6,364.19	-261.78	0.00	0.00	6,102.41	0.00	0.00	0.00	0.00	0.00	6,102.41
CVX	26,170.43	-2,088.69	0.00	0.00	24,081.74	0.00	0.00	0.00	0.00	0.00	24,081.74
DAL	1,965.75	-14,987.83	0.00	0.00	-13,022.08	0.00	0.00	0.00	0.00	0.00	-13,022.08
EQR	1,095.01	-8,608.14	0.00	0.00	-7,513.13	0.00	0.00	0.00	0.00	0.00	-7,513.13
IBM	24,026.10	-6,130.29	0.00	0.00	17,895.81	0.00	0.00	0.00	0.00	0.00	17,895.81
JNJ	166.72	-5,666.82	0.00	0.00	-5,500.10	0.00	0.00	0.00	0.00	0.00	-5,500.10
JPM	7,316.63	-48,469.11	0.00	0.00	-41,152.48	0.00	0.00	0.00	0.00	0.00	-41,152.48
NGG	1,003.08	-3,494.34	0.00	0.00	-2,491.26	0.00	0.00	0.00	0.00	0.00	-2,491.26
OMC	9,317.64	-8,736.15	0.00	0.00	581.48	0.00	0.00	0.00	0.00	0.00	581.48
RTN	241.91	-4,222.69	0.00	0.00	-3,980.78	0.00	0.00	0.00	0.00	0.00	-3,980.78
TXN	9,896.03	-3,833.23	0.00	0.00	6,062.80	0.00	0.00	0.00	0.00	0.00	6,062.80
VE	9,989.49	-38,586.27	0.00	0.00	-28,596.78	0.00	0.00	0.00	0.00	0.00	-28,596.78
WMB	24,607.03	0.00	0.00	0.00	24,607.03	0.00	0.00	0.00	0.00	0.00	24,607.03
WMT	4,497.20	-1,117.91	0.00	0.00	3,379.29	0.00	0.00	0.00	0.00	0.00	3,379.29
Total (Stocks)	**127,246.59**	**-149,111.72**	**0.00**	**0.00**	**-21,865.13**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**-21,865.13**
Total (All Asset Classes)	**127,246.59**	**-149,111.72**	**0.00**	**0.00**	**-21,865.13**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**-21,865.13**

FIGURE 1.5: Portfolio performance

[2] In Interactive Broker

RESULTS

We construct a figure to compare the change of the weights of each asset (figure 1.6).

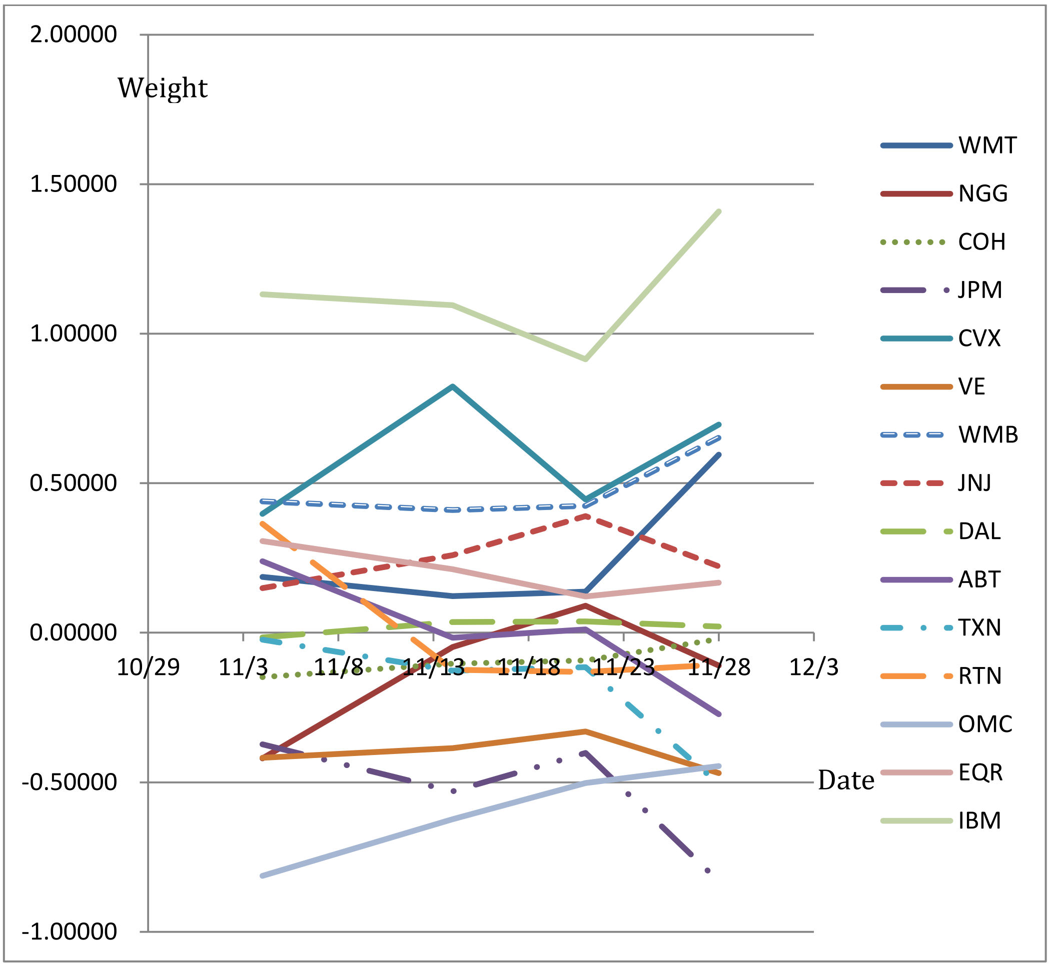

FIGURE 1.6: Change of the weights of each asset

From figure above, we conclude:

WMT: The position of WMT does not change much in the first three weeks, but its weight becomes larger in the last week.

NGG: From the first week to the second week, short weights become much smaller, until the third week, the weight is positive; however, in the last week, the weight turns to negative.

COH: For the whole period, the short weight does not change much but at the last week, its short weight is less than the weight in weeks prior.

JPM: For the whole period, we keep short a large weight of the asset, in the last week we short even more of the asset.

CVX: For the whole period, we keep a long position with a large weight.

VE: For the whole period, we keep short a large weight of this asset and the weight remains fairly constant.

WMB: For the whole period, we maintain a long position and the weight does not change significantly for the first three weeks but in the fourth week we increase the weight.

JNJ: For the whole period, we keep a long position, and for the first three weeks the weight is increasing, but in the last week, it decreased.

DAL: For the first week, we form a short position, but for the subsequent 3 weeks, we form a long position. The weight does not change much.

ABT: For the holding period, the weight is decreasing. For the first week we need to form long position in it, but in the subsequent 3 weeks, we need to short it more and more.

TXN: For the whole period, we keep a short position and the weight does not change much.

RTN: For the first week, we form a long position on it, but with the next 3 weeks, the weight becomes negative which means we need to short it.

TXN: For the whole period, we keep a short position and the short weight is very large.

EQR: For the whole period, we keep a long position and the weight is slightly decreasing.

IBM: For the whole period, we keep a long position and the weight is very large.

Our optimal portfolio is based on the past performance of assets. Thus, we can conclude that WMT, CVX, WMB, JNJ, EQR, IBM performed well in the past while CVX, JPM, VE, TXN, OMC performed poorly in the past.

Limitation

We find that our portfolio realized a loss. We notice that there are limitations in our optimization model. First, it based on the past performance, so it is sensitive to the length of time that we choose. That is why we choose our length of time to be 6 months.

Secondly, since the model is based on the past performance of the assets, it does not represent their future performances. There is always news which is unpredictable to the market and affects the market, such as the instability of Italy, which has a large effect the on the equity market, leading to the market's slumping.

Finally, the portfolio is based on the total risk. Next part we will use Capital Asset Pricing Model to analyze and improve our portfolio.

II. CAPITAL ASSET PRICING MODEL ANALYSIS

Our portfolio we formed in the first part is based on the total risk, which include systematic risk and unsystematic risk, which is

Total Risk = Systematic Risk + Unsystematic Risk

The unsystematic risk can be reduced by diversification. Capital market theory says that equilibrium security returns depends on a stock's or a portfolio's systematic risk. Thus we use the CAPM to analyze our portfolio, including the total risk, systematic risk and unsystematic risk (Ruppert, 2006, p. 234).

METHOD

For holding period t, Let $R_{j,t}$ denote the return of the risky asset, $R_{M,t}$ denote the return of the market portfolio and $\mu_{f,t}$ denote the risk free rate.

According to the CAPM (Ruppert, 2006, p. 225-234), we have formula

$$R_{j,t} - \mu_{f,t} = \beta_j * (R_{M,t} - \mu_{f,t}) + \varepsilon_{j,t}$$

$R_{j,t} - \mu_{f,t}$ is called the excess return of asset j, $R_{M,t} - \mu_{f,t}$ is called the market risk premium, β refers to the sensitivity of an asset's return to the return on market index in the context of the market model.

β=covariance of asset j's return with the market return/variance of the market return

Now we let $R^*_{j,t} = R_{j,t} - \mu_{f,t}$, and let $R^*_{M,t} = R_{M,t} - \mu_{f,t}$. We expand to:

$$R^*_{j,t} = \alpha_j + \beta_j * R^*_{M,t} + \varepsilon_{j,t}$$

Here α_j is the intercept of the asset j, it can be used to test validity of the CAPM. $\varepsilon_{j,t}$ is white noise, which is uncorrelated with different assets (Rupert, 2006, p. 232-234).

PARAMETER ESTIMATION

The first step is to estimate the value of β and α. We can do this by regression. Let $R^*_{j,t}$ be the responsible variable and $R^*_{M,t}$ be the predictor variable. We regress $R^*_{j,t}$ onto $R^*_{M,t}$ (Ruppert, 2006, p. 169-172).

We choose the last three months' data which is form 2011/8/15 to 2011/11/11 to estimate our parameters. Note that the returns are daily. For the risk-free rate, we choose the one year T-bill rate. For the market portfolio, we choose the S&P 500. Note that here we have a total of 63 observations.

The formula to calculate β is

$$\beta_j = cov(R^*_{j,t}, R^*_{M,t})/var(R^*_{M,t})$$

Having estimated the values of β, we can compute the total risk for each asset. The formula for calculating total risk for each asset is

$$\sigma_j = \sqrt{\beta_j^2 * \sigma_m^2 + \sigma_{\epsilon,j}^2}$$

where σ_j is the total risk of asset j, σ_m is market risk, $\sigma_{\epsilon,j}^2$ is the unique risk of asset j.

Using the MATLAB, our results are (Table 2.1):

TABLE 2.1: Computing results

	Value of β	Value of α	Total risk
WMT	0.38070	0.00240	0.00014
NGG	0.41307	-0.00006	0.00019
COH	1.43155	0.00183	0.00109
JPM	1.680107	-0.00244	0.00129
CVX	1.04246	0.00060	0.00047
VE	1.93948	-0.00528	0.00194
WMB	1.37004	0.00069	0.00090
JNJ	0.58687	-0.00017	0.00016
DAL	1.22046	0.00083	0.00135
ABT	0.53802	0.00112	0.00017
TXN	0.95489	0.00159	0.00050
RTN	0.79345	0.00116	0.00035
OMC	1.14758	0.00040	0.00057
EQR	1.12166	-0.00111	0.00066
IBM	0.74221	0.00078	0.00031

The portfolio beta is a weighted beta, and is calculated from the formula

$$\beta = \sum_{j=1}^{N} (W_j * \beta j)$$

where W_j denotes the optimal weight we formed above in asset j, N is the number of assets. Here N=15. Note that here W is the weight we formed on November 14th.

Using MATLAB, the value of portfolio β is 0.30591.

Next, we need to estimate the portfolio 'epsilon' at different time t. It is calculated from the formula

$$\text{Portfolio epsilon} = \sum_{j=1}^{N} (Wj * \varepsilon_{j,t})$$

The plot 63 portfolio epsilon is (Figure 2.1):

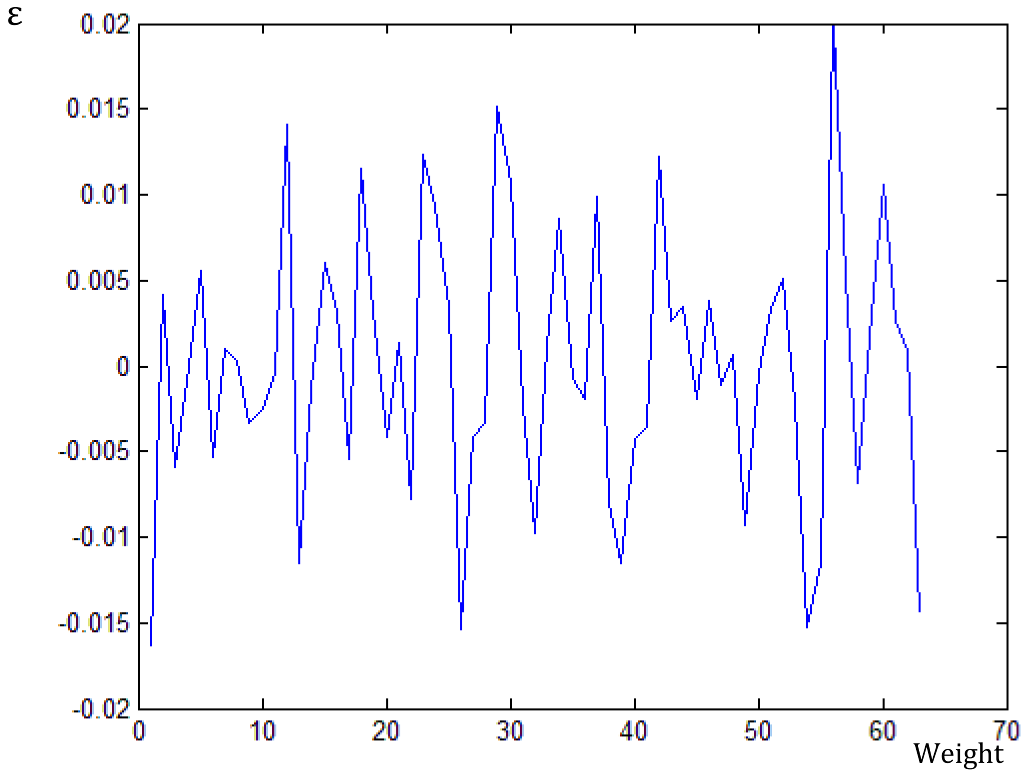

FIGURE 2.1: Portfolio epsilon

We can calculate the variance of the portfolio return, which denote the total risk of our portfolio. The formula is:

$$\mathrm{Var}(R_p) = \left(\sum_{j=1}^{N}\left(W_j * \beta_j\right)\right)^2 * \sigma_m{}^2 + \sum_{j=1}^{N}\left(W_j^2 * \sigma_{\varepsilon,j}^2\right)$$

$\left(\sum_{j=1}^{N}\left(W_j * \beta_j\right)\right)^2 * \sigma_m{}^2$ is called the market component, $\sum_{j=1}^{N}\left(W_j^2 * \sigma_{\varepsilon,j}^2\right)$ is called unique component. [6]

Using MATLAB, the total variance of our portfolio is 3.41752e-005. The market component accounts for 3.41567e-005 and the unique component accounts for 1.84689e-008. Thus the unique component accounts for 0.054% of the total variance.

RESULTS

According to the estimates, the portfolio β is around 0.306, which is below 1. Thus, the portfolio is non-aggressive, which means that my portfolio is not expected to outperform the market portfolio.

However, the individual assets COH, JPM, CVX, VE, WMB, DAL, OMC, EQR, of which the βs are all greater than 1, are aggressive.

COH is a company that produces upscale leather goods, so its products are not necessities. We know that such kind of products are elastic than other necessary goods in demand. Its large β explain its close relation with the market.

JPM is a financial company, CVX and WMB are Energy Companies, and their large β is understandable.

VE is an environmental company; it has large β maybe because environmental protection is hot in the modern world.

DAL is Airline Company, which has close relation with market. For example, if oil prices rise up, the ticket of flight also rise up.

OMC is advertising company and EQR is Real Estate Company, they both have close relation with the market.

Among all the companies, WMT has the smallest β; this is understandable because WMT is a retail store. Most of its products are necessities, so the demand for their products is inelastic and has little relation with the market.

The total risk of most assets is small. Some assets, however, have relatively larger risk, such as COH, JPM, VE, WMB, and DAL. We find that these assets also have larger βs which are greater than 1, that is why COH, JPM, VE, WMB, and DAL have relatively larger total risk. We know that total risk includes systematic risk and unsystematic risk while β measures the systematic risk.

We can also find that the unique component of the total variance of the portfolio accounts for 0.054% of the total variance, which is very small. Thus, our portfolio is well diversified.

The portfolio weights plot is as follows (Figure 2.2).

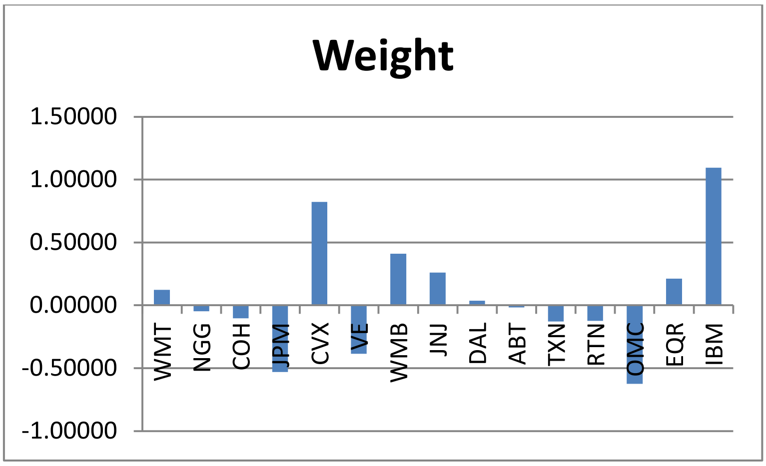

FIGURE 2.2: Portfolio weights

We can construct a table to compare the estimated values (Table 2.2).

TABLE 2.2: Estimated values

	Value of β	Value of α	Total risk	Weights
WMT	0.38070	0.00240	0.00014	0.12234
NGG	0.41307	-0.00006	0.00019	-0.04750
COH	1.43155	0.00183	0.00109	-0.10337
JPM	1.680107	-0.00244	0.00129	-0.53042
CVX	1.04246	0.00060	0.00047	0.82382
VE	1.93948	-0.00528	0.00194	-0.38629
WMB	1.37004	0.00069	0.00090	0.40986
JNJ	0.58687	-0.00017	0.00016	0.25938
DAL	1.22046	0.00083	0.00135	0.03580
ABT	0.53802	0.00112	0.00017	-0.01673
TXN	0.95489	0.00159	0.00050	-0.12745
RTN	0.79345	0.00116	0.00035	-0.12323
OMC	1.14758	0.00040	0.00057	-0.62316
EQR	1.12166	-0.00111	0.00066	0.21191
IBM	0.74221	0.00078	0.00031	1.09506

One interesting finding is that COH, JPM, VE, OMC have β>1, meanwhile they have negative weights, especially for JPM, as it has a large shorting weight.

From the values of α, we can see that some values are greater than 0, some are smaller than 0. Thus the CAPM here is not very accurate. For those assets whose α>0, they are underpriced. Those assets are WMT, COH, CVX, WMB, DAL, ABT, TXN, RTN, OMC, and IBM. Other assets are overpriced. So we might long the assets which are underpriced and short the assets which are overpriced.

III. FACTOR MODEL

Now we have implemented the CAPM to simplify the estimation of expectations and covariance of assets returns. However, using CAPM for this purpose is dangerous since the estimates depend on the validity of CAPM. We use a more realistic factor model to estimate expectations, variances and covariances of asset returns.

A factor model (Ruppert, 2006, p.242-245) is mathematical profile measuring the extent a portfolio of stocks is influenced by a range of economic factors, such as changes in interest rates, inflation, and GDP growth rates.

The objective of this part is to see how factor models affect our estimates through estimating the expected return of assets in the future using past data. We assume we are on January 1st, 2011. We need to form a portfolio for a holding period of 10 months, which means the portfolio will pass through October 31st. We will refer to seven factor models. By implementing each of these models we estimate the expected returns, variances and covariances of the asset returns. By comparing the computed results with the actual results, we will try to find the relationship between asset returns and each factor.

FACTORS

We refer to 5 factors:

i) Excess return of market portfolio, where the market portfolio is the S&P 500. The risk free rate is the 1-year T-bill rate. This factor is the CAPM part.
ii) SML, which means "small minus large". It is the difference in returns on a portfolio of small stocks and a portfolio on large stocks.
iii) HML, which means "high minus low". It is the difference in returns on a portfolio of high book-to-market value stocks and a portfolio of low book-to-market value stocks (Ruppert, 2009, p.245).
iv) Effective Federal Funds Rate. It is the interest rate at which depository institutions actively trade balances held at the Federal Reserve, called federal funds, with each other, usually overnight, on an uncollateralized basis. Institutions with surplus balances in their accounts lend those balances to institutions in need of larger balances. It is a market-determined rate. The U.S. Federal Reserve targets the federal funds rate to manage the money supply and to keep inflation low, promoting economic growth and full employment. Thus the federal funds rate is an important benchmark in financial markets. (The Washington Post, 2004)
v) Return of the investment in gold. Gold is regarded as an inflation barometer and many investors choose it as hedging strategy. (Investopedia, 2009)

MODELS

There are 7 factor models.

i) The French and Fama Model (Ruppert, 2006, p.242-246).
This model includes 3 factors, which are the excess return of market portfolio, SML and HML.
ii) A two factor model with two factors which are the CAPM excess return and the effective federal funds rate.
iii) A two factor model with two factors which are the CAPM excess return and the return of the investment in gold.
iv) Extending the French and Fama Model by adding the factor effective federal funds rate. It is a four factor model.
v) Extending the French and Fama Model by adding the factor return of the investment in gold. It is a four factor model.
vi) A three factor model with factors which are excess return of market portfolio, effective federal funds rate and return of the investment in gold.
vii) Extending the French and Fama Model by adding the factor return of the investment in gold and the factor return of the investment in gold. It is a five factor model.

METHOD

The formula for the factor model is

$$R_{j,t} = \beta_{0,j} + \beta_{1,j} * F_{1,t} + \beta_{2,j} * F_{2,t} + \cdots + \beta_{p,j} * F_{p,t} + \varepsilon_{j,t}$$

$R_{j,t}$ is the excess return over one year T-bill rate of asset j at time t, $F_{1,t}$, $F_{2,t}$, …, $F_{p,t}$ are p factors which are independent of the assets. The $\beta_{1,j}$, $\beta_{2,j}$,…, $\beta_{p,j}$ are called factor loadings. $\beta_{i,j}$ represents the sensitivity of the j^{th} return to the i^{th} factor. $\varepsilon_{1,t}$, $\varepsilon_{2,t}$, …, $\varepsilon_{n,t}$are white noise called the unique risk of the individual assets, and they are uncorrelated.

The first step is to estimate the factor loadings $\beta_{1,j}$, $\beta_{2,j}$,…, $\beta_{p,j}$. Here we fit factor models using time series regression. Take $F_{1,t}$, $F_{2,t}$, …, $F_{p,t}$ as predictor variables, $R_{j,t}$ as the responsible variable. For each factor model, we regress each asset onto the factors. We refer to the computed results by each factor model.

Having the factor loadings, the second step is to use the estimated parameters to estimate expected returns of assets for a holding period of 10 months. We know

$$R_{j,t} = \beta_{0,j} + \beta_{1,j} * F_{1,t} + \beta_{2,j} * F_{2,t} + \cdots + \beta_{p,j} * F_{p,t} + \varepsilon_{j,t}$$

Taking the expectation, we get

$$E(R_{j,t}) = \beta_{0,j} + \beta_{1,j} * E(F_{1,t}) + \beta_{2,j} * E(F_{2,t}) + \cdots + \beta_{p,j} * E(F_{p,t})$$

Now we already have the estimated values $\beta_{1,j}$, $\beta_{2,j}$,…, $\beta_{p,j}$. As for the $E(F_{1,t})$, $E(F_{2,t})$, … , $E(F_{p,t})$, we use the arithmetic mean value. Thus we can get the excess return of each asset;

we need to add back the risk free rate since we need the expected returns of each asset. Then we must compound the return by 210 days since what we get now is only the expected daily return of each asset. To get the expected return of holding period of 10 months, we need to compound it by 210 days.

Finally we use the computed results and our portfolio optimization part to form our optimal tangency portfolio. After holding period of 10 months, we can compare the theoretical portfolio return and actual return and find which model performs the best.

Next we will display the computed results for each factor model using MATLAB

i) French and Fama Model
The factor loading table is (Table 3.1):

TABLE 3.1: French and Fama Model factors

	Excess return of market portfolio	SML	HML
WMT	0.00617	-0.00233	-0.00639
NGG	0.00923	-0.00227	-0.0035
COH	0.01278	0.00452	-0.00329
JPM	0.00912	-0.00646	0.01624
CVX	0.01008	-0.00279	0.0003
VE	0.01588	-0.00518	-0.00335
WMB	0.01257	0.00029	0.00516
JNJ	0.00732	-0.00329	-0.00634
DAL	0.01325	-0.00144	0.00157
ABT	0.00643	-0.00301	-0.00261
TXN	0.01054	0.00037	-0.00135
RTN	0.00909	-0.00069	-0.00239
OMC	0.01061	0.00241	-0.00058
EQR	0.01192	0.00132	0.00491
IBM	0.00918	-0.00116	-0.00486

We can see that the portfolio loads relatively highly on the first factor.

ii) A model with two factors which are CAPM excess return and the effective federal funds rate.
The factor loading table is (Table 3.2):

TABLE 3.2: CAPM excess return and the effective federal funds rate

	Excess return of market portfolio	Effective federal fund rate
WMT	0.39	312.28
NGG	0.79	103.51
COH	1.31	750.14
JPM	1.34	-765.53
CVX	0.99	295.04
VE	1.43	-659.69
WMB	1.45	-720.1
JNJ	0.49	-69.07
DAL	1.39	-1566.66
ABT	0.52	-623.44
TXN	1.06	1633.8
RTN	0.86	-658.34
OMC	1.12	915.43
EQR	1.42	320.36
IBM	0.77	617.9

We can see that in this model, the excess return of the assets load extremely highly on the factor of effective federal fund rate. They also load highly on the excess return of market portfolio; however, compared to the effective federal fund rate, the effect is very subtle.

iii) A model with two factors which are CAPM excess return and return of the investment in gold.
The factor loading table is (Table 3.3):

TABLE 3.3: CAPM excess return and return of the investment in gold

	Excess return of market portfolio	Return of gold
WMT	0.39413	-0.03154
NGG	0.77853	0.03986
COH	1.28629	0.10926
JPM	1.33431	0.01864
CVX	0.97987	0.07457
VE	1.43348	-0.03632
WMB	1.38981	0.30036
JNJ	0.49397	-0.02242
DAL	1.36473	0.11274
ABT	0.52226	-0.00981
TXN	1.07145	-0.05095
RTN	0.87007	-0.06629
OMC	1.12171	0.01411
EQR	1.44978	-0.14645
IBM	0.77157	-0.00011

We can see that the excess return of portfolio loads highly on the factor of excess return of market portfolio while loads slightly on the factor of return of gold.

iv) Extending French and Fama Model by adding the factor of the effective federal fund rate. The factor loading table is (Table 3.4):

TABLE 3.4: French and Fama Model factors and the effective federal fund rate

	Excess return of market portfolio	SML	HML	Effective federal fund rate
WMT	0.00617	-0.00233	-0.0064	-17.28
NGG	0.00924	-0.00227	-0.00352	-47.19
COH	0.01265	0.00456	-0.00292	665.61
JPM	0.00908	-0.00644	0.01636	212.76
CVX	0.01001	-0.00277	0.00051	379.99
VE	0.01604	-0.00523	-0.00378	-786.38
WMB	0.01265	0.00027	0.00496	-367.46
JNJ	0.0074	-0.00331	-0.00656	-401.51
DAL	0.01354	-0.00152	0.000775	-1444.14
ABT	0.00658	-0.00305	-0.00302	-759.93
TXN	0.0102	0.00046	-0.00043	1672.85
RTN	0.00925	-0.00073	-0.00281	-761.56
OMC	0.01041	0.00246	-0.00005	977.65
EQR	0.01178	0.00135	0.00529	694.61
IBM	0.0091	-0.00114	-0.00464	407.41

In this model, the response variable loads extremely heavily on the factor of the effective federal fund rate.

v) Extending the French and Fama Model by adding the factor Return of the investment in gold.
The factor loading table is (Table 3.5):

TABLE 3.5: French and Fama Model factors and Return of the investment in gold

	Excess return of market portfolio	SML	HML	Return of gold
WMT	0.0062	-0.00229	-0.00636	-0.02393
NGG	0.00919	-0.00231	-0.00353	0.02742
COH	0.01271	0.00444	-0.00335	0.05064
JPM	0.00914	-0.00643	0.01626	-0.01556
CVX	0.01	-0.00287	0.00023	0.05599
VE	0.01596	-0.0051	-0.00328	-0.05562
WMB	0.01222	-0.0001	0.00487	0.24306
JNJ	0.00734	-0.00327	-0.00632	-0.01323
DAL	0.01314	-0.00156	0.00148	0.07265
ABT	0.00644	-0.003	-0.0026	-0.00882
TXN	0.01067	0.00051	-0.00125	-0.08521
RTN	0.00922	-0.00055	-0.00229	-0.08713
OMC	0.01066	0.00246	-0.00054	-0.03509
EQR	0.01223	0.00165	0.00515	-0.20952
IBM	0.0092	-0.00114	-0.00485	-0.01268

We can see that the portfolio excess returns load more heavily on the factor of return gold compared to the other factors.

vi) Extending the CAPM by adding the two factors of the effective federal fund rate and the return of asset gold

The factor loading table is:

TABLE 3.6: The CAPM model factor, the effective federal fund rate and the return of asset gold

	Excess return of market portfolio	Effective federal fund rate	Return of gold
WMT	0.39356	324.29046	-0.03325
NGG	0.77838	89.28093	0.03939
COH	1.28505	712.0351	0.10551
JPM	1.33567	-773.73241	0.02271
CVX	0.97939	268.62491	0.07316
VE	1.43462	-647.80762	-0.03291
WMB	1.39127	-830.13881	0.30473
JNJ	0.49408	-61.09078	-0.0221
DAL	1.36756	-1610.42936	0.12121
ABT	0.52335	-621.08234	-0.00654
TXN	1.06854	1655.34545	-0.05965
RTN	0.87119	-635.60724	-0.06295
OMC	1.1201	912.06536	0.00932
EQR	1.44913	373.95742	-0.14841
IBM	0.77048	619.11666	-0.00337

In this model, the response variable loads highly on the factor of the excess return of the market portfolio, extremely heavily on the factor of the effective federal fund rate and slightly on the return of gold.

vii) Extending the French and Fama Model by adding two factors of the effective federal fund rate and the return of asset gold
The factor loading table is:

TABLE 3.7: French and Fama Model factors, effective federal fund rate and the return of asset gold

	Excess return of market portfolio	SML	HML	Effective federal fund rate	Return of gold
WMT	0.0062	-0.00229	-0.00636	-6.61	-0.02389
NGG	0.0092	-0.00231	-0.00356	-59.61	0.02781
COH	0.01258	0.00448	-0.00299	644.86	0.04648
JPM	0.0091	-0.00642	0.01638	220.34	-0.01698
CVX	0.00993	-0.00285	0.00043	356.02	0.05369
VE	0.01611	-0.00514	-0.00371	-763.75	-0.0507
WMB	0.01231	-0.00013	0.00461	-477.35	0.24614
JNJ	0.00742	-0.0033	-0.00654	-396.74	-0.01067
DAL	0.01343	-0.00165	0.00066	-1480.83	0.08219
ABT	0.00659	-0.00304	-0.00302	-758.18	-0.00393
TXN	0.01033	0.00061	-0.00029	1715.82	-0.09627
RTN	0.00936	-0.0006	-0.00269	-724.75	-0.08246
OMC	0.01047	0.00253	0.00001	996.18	-0.04151
EQR	0.01207	0.0017	0.00559	790.42	-0.21461
IBM	0.00912	-0.00111	-0.00462	414.27	-0.01535

In this model, the response variable loads extremely heavily on the factor of the effective federal fund rate compared to the other 3 factors.

For the 7 models, we find that the estimated expected returns of each asset, variance of return of each asset and are almost the same. We construct a table of estimates of expected return, variance of return, actual return and actual variance of each asset. We note that the expected returns of each model are almost the same, and the variance of returns of each model are close to each other, so we just list expected returns and the variance of returns of one model.

The table of expected return is (Table 3.7):

TABLE 3.7: Expected return and actual return

Return / Assets	Expected return	Actual return
WMT	0.03489	0.07384
NGG	-0.08479	0.17095
COH	0.49209	0.18871
JPM	0.05975	-0.16281
CVX	0.20901	0.17822
VE	-0.04741	-0.48865
WMB	0.21820	0.24114
JNJ	0.00160	0.06907
DAL	0.18507	-0.32381
ABT	-0.06178	0.16678
TXN	0.25633	-0.03818
RTN	-0.05099	-0.01163
OMC	0.18805	-0.01221
EQR	0.53475	0.14991
IBM	0.13198	0.27419

The table of variance is (Table 3.8):

TABLE 3.8: Estimated variance and actual variance

Variance / Assets	Estimated variance	Actual variance
WMT	0.00008	0.00011
NGG	0.00024	0.00019
COH	0.00043	0.00062
JPM	0.00038	0.00058
CVX	0.00017	0.00028
VE	0.00044	0.00117
WMB	0.00044	0.00065
JNJ	0.00007	0.00012
DAL	0.00082	0.00101
ABT	0.00008	0.00013
TXN	0.00028	0.00029
RTN	0.00017	0.00022
OMC	0.00024	0.00033
EQR	0.00042	0.00036
IBM	0.00013	0.00019

Having estimated the expected return, we can use our portfolio optimization material to form our optimal portfolio. The table of optimal portfolio weights is given below:

TABLE 3.9: Optimal portfolio weights for 7 models

	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII
WMT	0.30478	0.43081	0.43728	0.43113	0.4357	0.42954	0.42999
NGG	0.04045	-0.18313	-0.17418	-0.19646	-0.18906	-0.18169	-0.19409
COH	-0.05443	0.2587	0.25596	0.2372	0.23137	0.25921	0.23637
JPM	0.00998	-0.2571	-0.26353	-0.20393	-0.19844	-0.25645	-0.20486
CVX	0.00144	0.40528	0.40836	0.45318	0.45831	0.41071	0.45757
VE	-0.09024	-0.37168	-0.37095	-0.37788	-0.38047	-0.37135	-0.37786
WMB	-0.06184	-0.06822	-0.08106	-0.07006	-0.07608	-0.06764	-0.06613
JNJ	0.44123	0.33096	0.32772	0.34115	0.33097	0.33014	0.34229
DAL	-0.01844	-0.02052	-0.02702	-0.02017	-0.02523	-0.0202	-0.01968
ABT	0.33759	-0.06787	-0.08646	-0.06012	-0.06976	-0.06646	-0.05724
TXN	-0.01232	0.15621	0.17263	0.14874	0.15892	0.15399	0.14534
RTN	0.08448	-0.35619	-0.36614	-0.36692	-0.37748	-0.35873	-0.37181
OMC	-0.02056	0.02937	0.0503	-0.00932	0.00593	0.02836	-0.01213
EQR	-0.06703	0.36251	0.35103	0.38245	0.37555	0.36167	0.38326
IBM	0.10491	0.35089	0.36608	0.311	0.31978	0.34889	0.30899

RESULTS

Based on our computed results, we can conclude that our portfolio is extremely sensitive to the effective federal fund rate factor, which is understandable because the federal fund rate is the primary way for the United States government to regulate and control the country's economy.

We also find that for the 7 models, the expected returns of each model are almost the same, and the variance of returns of each model are close to each other, so we just list expected returns and the variance of returns of one model.

As for the covariance matrix of each model, we find that the difference between French and Fama Model with those models extending French and Fama Model is smaller, and that the difference between the models which extend CAPM is smaller, but the difference between the French and Fama Model and the CAPM is larger. We calculate the difference of the covariance matrix of each model and get the Euclidean norm of them to see their differences. To calculate the differences, assume the covariance matrix of one model is Cov_1, the covariance matrix of another model is Cov_2.

$$Cov_1 = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$Cov_1 = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b & \dots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix}$$

$$Cov_1 - Cov_2 = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \dots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \dots & a_{2n} - b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} - b_{n1} & a_{n2} - b_{n2} & \dots & a_{nn} - b_{nn} \end{bmatrix}$$

Then we calculate the value

$$D = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij} - b_{ij})^2}$$

to see their difference of matrixes of different models.

The results are listed in the table as follows.

TABLE 3.9: Value of Euclidean Norm

	Value of Norm
Model I & II	5.91359E-05
Model I & III	6.11056E-05
Model I & IV	1.11363E-05
Model I & V	1.04423E-05
Model I & VI	6.00092E-05
Model I & VII	1.66867E-05
Model II & III	1.58109E-05
Model II & IV	0.000059454
Model II & V	5.87119E-05
Model II & VI	1.17299E-05
Model II & VII	5.92872E-05
Model III & IV	0.000063046
Model III & V	5.92186E-05
Model III & VI	1.22649E-05
Model III & VII	6.14921E-05
Model IV & V	1.43004E-05
Model IV & VI	6.01582E-05
Model IV & VII	1.08445E-05
Model V & VI	5.77566E-05
Model V & VII	0.000011539
Model VI & VII	0.000058117

As for the expected portfolio return and actual return, we construct a table of the comparisons between them to see the performance of each model (Table 3.10).

TABLE 3.10: Comparisons between expected portfolio return and actual return

	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII
Expected Return	0.53475	0.53475	0.53475	0.53475	0.53475	0.53475	0.53475
Actual Return	0.48493	0.49514	0.49523	0.48611	0.4867	0.49576	0.48834
Difference (%)	10.27	8	7.98	10	9.87	7.87	9.51

From this table, we find that Model VI performs the best because it has the smallest difference between actual return and the expected return. This Model is the extended CAPM with the factors effective federal fund rate and return of gold. Models II and III also perform well compared to the other models. Model II is the extended CAPM with the effective federal fund rate factor, and Model III is the extended CAPM with the factor return of gold. The French and Fama Model performs the worst compared to the other models. Thus we conclude that the models which extend CAPM perform better than the French and Fama Model and its extensions.

Appendix A

APPENDIX A: MATLAB CODE

optimalPortfolio.m

```
function[optimalWeights] =
optimalPortfolio(expReturns,CovMatrix,expPortfolioReturn)
n=length(expReturns);
I=zeros(n,1);
for ii=1:n
    I(ii)=1;
end
A=expReturns'*inv(CovMatrix)*I;
B=expReturns'*inv(CovMatrix)*expReturns;
C=I'*inv(CovMatrix)*I;
D=B*C-A*A;
g=(B*inv(CovMatrix)*I-A*inv(CovMatrix)*expReturns)/D;
h=(C*inv(CovMatrix)*expReturns-A*inv(CovMatrix)*I)/D;
W=g+h*expPortfolioReturn;
optimalWeights=W;
```

efficientFrontier.m

```
clear;clc
format long

%read the excel file to MATLAB
Data=xlsread('SixMongthsData3.xlsx');
[a n]=size(Data);

%calculate the number of rows of the weekly data
count=0;
for ii=1:5:a
    count=count+1;
end
m=count;
weeklyData=zeros(m,n);
for ii=1:m
    for jj=1:n
        weeklyData(ii,jj)=Data(5*ii-4,jj);
    end
end

%calculate the return of holding period of one week
Return=zeros(m-1,n);
for ii=1:m-1
    for jj=1:n
        Return(ii,jj)=weeklyData(ii,jj)/weeklyData(ii+1,jj)-1;
    end
end
```

```
expReturns=zeros(n,1);
for ii=1:n
    expReturns(ii)=mean(Return(:,ii));
end
CovMatrix=cov(Return);
rf=0.11*0.01/52;
disp(weeklyData);

%Plot 50 pairs of Up and Sigma
Sigma=zeros(300,1);

%Calculate the min variance and the the expected return when the variance
%is at the minimum value
I=ones(n,1);

A=expReturns'*inv(CovMatrix)*I;
B=expReturns'*inv(CovMatrix)*expReturns;
C=I'*inv(CovMatrix)*I;
D=B*C-A*A;
g=(B*inv(CovMatrix)*I-A*inv(CovMatrix)*expReturns)/D;
h=(C*inv(CovMatrix)*expReturns-A*inv(CovMatrix)*I)/D;
Umin=-(g'*CovMatrix*h)/(h'*CovMatrix*h);
minVariance=g'*CovMatrix*g-(g'*CovMatrix*h)^2/(h'*CovMatrix*h);
Stdmin=sqrt(minVariance);

%Vary the expected return of the portfolio along the grid, the expected
%return of the portfolio is between the min and max expected return of 10
%assets
muP=linspace(min(expReturns),0.05,300);
omegaP=zeros(n,300);
for ii=1:300
    omegaP(:,ii)=optimalPortfolio(expReturns,CovMatrix,muP(ii));
    Sigma(ii)=sqrt(omegaP(:,ii)'*CovMatrix*omegaP(:,ii));
end

%find the tangency portfolio
sharperatio=(muP-rf)'./Sigma;
Itangency=find(sharperatio==max(sharperatio))
disp(max(sharperatio))
disp(omegaP(:,Itangency));

%plot the efficient frontier
imin=find(Sigma==min(Sigma));
Ieff=(muP>=muP(imin));
sharperatio=(muP-rf)'./Sigma;
Itangency=find(sharperatio==max(sharperatio));
plot(Sigma(Ieff),muP(Ieff),Sigma(Itangency),muP(Itangency),'*',Sigma(imin),
muP(imin),'o',0,rf,'x');
line([0 Sigma(Itangency)],[rf,muP(Itangency)]);

%plot the bar graph
bar(omegaP(:,Itangency));
```

ReturnRegression.m

```
clear;clc
format long
```

```
%read the excel file to MATLAB
Price=xlsread('Price.xlsx');

%convert the T-bill rate to daily rate
Rf=xlsread('Risk-free rate.xlsx')*0.01/253;

[m n]=size(Price);

%calculate the daily return
%the first 15 colunms are risky asset returns
%the last columns is the market portfolio returns
Return=zeros(m-1,n);
for ii=1:m-1
    Return(ii,:)=Price(ii,:)./Price(ii+1,:)-1;
end

%Rstar is the excess return,
Rstar=zeros(m-1,n);
for ii=1:m-1
    Rstar(ii,:)=Return(ii,:)-Rf(ii);
end

%do regression with each risky asset on the market portfolio, note that the
%last columns of the Rstar matrix is the market portfolio data
Beta=zeros(n-1,1);
Alfa=zeros(n-1,1);
for ii=1:n-1
    CovarianceMatrix=cov(Rstar(:,ii),Rstar(:,n));
    Beta(ii)=CovarianceMatrix(1,2)/var(Rstar(:,n));
    Alfa(ii)=mean(Rstar(:,ii))-Beta(ii)*mean(Rstar(:,n));
end
disp('Beta of each asset is: ');
disp(Beta)
disp('Alfa of each asset is:')
disp(Alfa);

%claculate the tatal risk
Error=zeros(n-1,1);
for ii=1:n-1
    estimate=Beta(ii)*Rstar(:,n)+Alfa(ii);
    Error(ii)=sum((Rstar(:,ii)-estimate).*(Rstar(:,ii)-estimate))/(m-2);
end
TotalRisk=(Beta.*Beta)*var(Rstar(:,n))+Error

%Calculate the portfolio Beta
%W is the weights as computed
W=[0.021618640922453;0.326641518164823;-
0.097690204496053;0.034985997273484;
-0.236020890794171;-0.159294493975107;0.215775653531656;0.587147883446866;
0.064550957415628;0.212653550440708;0.07857152788260;-0.134657676097017;
-0.217116369082454;0.057010676735491;0.245823228631088];
PortfolioBeta=W'*Beta

%calculate portfolio epsilon
Epsilon=zeros(m-1,n-1);
for ii=1:n-1
    Epsilon(:,ii)=Beta(ii)*Rstar(:,n)+Alfa(ii)-Rstar(:,ii);
end
PortfolioEpsilon=zeros(m-1,1);
```

```
for ii=1:m-1
    PortfolioEpsilon(ii)=Epsilon(ii,:)*W;
end
disp('Portfolio Epsilon is:');
disp(PortfolioEpsilon)

%calculate the portfolio unique component
PortfolioQunique=sum((W.*W).*(Error.*Error))

%calculate the market component
PortfolioMarket=PortfolioBeta^2*var(Rstar(:,n))

%portfolio risk
PortfolioVariance=(PortfolioQunique+PortfolioMarket)

PortfolioQunique/PortfolioVariance*100
```

FactorModel.m

```
clear;clc
R=xlsread('ExcessReturn.xlsx');
ResV=xlsread('Model I.xlsx');
[m n]=size(R);
[a b]=size(ResV);

%Regress excess return onto factors
X=zeros(a,b+1);
X(:,1)=1;
for ii=2:b+1
    X(:,ii)=ResV(:,ii-1);
end
Beta=zeros(b+1,n);
for ii=1:n
    Beta(:,ii)=inv(X'*X)*X'*R(:,ii);
end

Intercrept=zeros(n,1);
for ii=1:n
    Intercrept(ii)=Beta(1,ii);
end
for ii=1:n
    esR=X*Beta(:,ii);
    Epsilon(:,ii)=R(:,ii)-esR;
end

%calculate diagnol matrix of the residual sum of square errors form the
%regression
Sigma=zeros(n,1);
for ii=1:n
    Sigma(ii)=sum(Epsilon(:,ii).*Epsilon(:,ii))/(m-1-b);
end
DiaMatrixError=zeros(n,n);
for ii=1:n
    DiaMatrixError(ii,ii)=Sigma(ii);
end

Beta(1,:)=[];
```

```
disp(Beta)
disp(Intercrept)

%estimate the expected return for holding period of 10 months
eF=zeros(b,1);
for ii=1:b
    eF(ii)=mean(ResV(:,ii));
end
eRt=Intercrept+Beta'*eF;
%there are 210 openning days from 2011/1/1 to 2011/10/31
eRt=(1+eRt+0.27*0.01/252).^210-1

%estimate each asset's return variance
eVarRt=zeros(n,1);
for ii=1:n
    eVarRt(ii)=Beta(:,ii)'*cov(ResV)*Beta(:,ii)+Sigma(ii);
end
disp(eVarRt)

%estimate the covaraince matrix
CovMatrix=Beta'*cov(ResV)*Beta+DiaMatrixError

%use the estimates to form optimal tangency portfolios
rf=0.27*0.01*210/252;

%Plot 50 pairs of Up and Sigma
Sigma2=zeros(50,1);
I=ones(n,1);
A=eRt'*inv(CovMatrix)*I;
B=eRt'*inv(CovMatrix)*eRt;
C=I'*inv(CovMatrix)*I;
D=B*C-A*A;
g=(B*inv(CovMatrix)*I-A*inv(CovMatrix)*eRt)/D;
h=(C*inv(CovMatrix)*eRt-A*inv(CovMatrix)*I)/D;
Umin=-(g'*CovMatrix*h)/(h'*CovMatrix*h);
minVariance=g'*CovMatrix*g-(g'*CovMatrix*h)^2/(h'*CovMatrix*h);
Stdmin=sqrt(minVariance);

%Vary the expected return of the portfolio along the grid, the expected
%return of the portfolio is between the min and max expected return of 10
%assets
muP=linspace(min(eRt),max(eRt),50);
omegaP=zeros(n,50);
for ii=1:50
    omegaP(:,ii)=optimalPortfolio(eRt,CovMatrix,muP(ii));
    Sigma2(ii)=sqrt(omegaP(:,ii)'*CovMatrix*omegaP(:,ii));
end

%find the tangency portfolio
sharperatio=(muP-rf)'./Sigma2;
Itangency=find(sharperatio==max(sharperatio))
disp(max(sharperatio))
disp(omegaP(:,Itangency));
PortR=omegaP(:,Itangency)'*eRt

%compute actual return, variance and covariance of the actual return
AcReturn=xlsread('actualReturn.xlsx');
[A B]=size(AcReturn);
for ii=1:n
```

```
    AcVar(ii)=var(AcReturn(:,ii));
end
AcCov=cov(AcReturn)
disp(AcVar)
acR=[0.07384;0.17095;0.18871;-0.16281;0.17822;-0.48865;0.24114;0.06907;
-0.32381;0.16678;-0.03818;-0.01163;-0.01221;0.14991;0.27419]

acPortR=omegaP(:,Itangency)'*acR
```

Test.m

```
clear;clc

R=xlsread('ExcessReturn.xlsx');
ResV=xlsread('Model VII.xlsx');
[m n]=size(R);
[a b]=size(ResV);

%Regress excess return onto factors
X=zeros(a,b+1);
X(:,1)=1;
for ii=2:b+1
    X(:,ii)=ResV(:,ii-1);
end
Beta=zeros(b+1,n);
for ii=1:n
    Beta(:,ii)=inv(X'*X)*X'*R(:,ii);
end

%estimate error
for ii=1:n
    esR=X*Beta(:,ii);
    Epsilon(:,ii)=R(:,ii)-esR;
end

%calculate diagnol matrix of the residual sum of square errors form the
%regression
Sigma=zeros(n,1);
for ii=1:n
    Sigma(ii)=sum(Epsilon(:,ii).*Epsilon(:,ii))/(m-1-b);
end
DiaMatrixError=zeros(n,n);
for ii=1:n
    DiaMatrixError(ii,ii)=Sigma(ii);
end
AIC=zeros(n,1);
for ii=1:n
    AIC(ii)=m*log(Sigma(ii))+2*(b+1);
end
```

Appendix B

APPENDIX B: COVARIANCE MATRIX

Model I

	WMT	NGG	COH	JPM	CVX
WMT	0.00007796	0.00004246	0.00006529	0.00005405	0.00004931
NGG	0.00004246	0.00024143	0.00013395	0.00013081	0.00010212
COH	0.00006529	0.00013395	0.00043303	0.00020848	0.00016515
JPM	0.00005405	0.00013081	0.00020848	0.00037594	0.00017456
CVX	0.00004931	0.00010212	0.00016515	0.00017456	0.00017223
VE	0.00007419	0.00014838	0.00023711	0.00024339	0.00018405
WMB	0.00006774	0.00014806	0.00024874	0.00026259	0.00018853
JNJ	0.00002992	0.00005298	0.00008098	0.00007286	0.0000626
DAL	0.00006722	0.00014166	0.00023407	0.00024292	0.00017841
ABT	0.00002844	0.00005478	0.00008515	0.00008691	0.00006718
TXN	0.00005255	0.00010811	0.00018111	0.00017597	0.00013442
RTN	0.00004368	0.00008782	0.00014494	0.00014011	0.00010845
OMC	0.00005491	0.00011484	0.0001965	0.00018724	0.00014315
EQR	0.00006495	0.00014235	0.0002412	0.00025113	0.00018114
IBM	0.00004193	0.00008082	0.00013193	0.00012142	0.00009818

	VE	WMB	JNJ	DAL	ABT
WMT	0.00007419	0.00006774	0.00002992	0.00006722	0.00002844
NGG	0.00014838	0.00014806	0.00005298	0.00014166	0.00005478
COH	0.00023711	0.00024874	0.00008098	0.00023407	0.00008515
JPM	0.00024339	0.00026259	0.00007286	0.00024292	0.00008691
CVX	0.00018405	0.00018853	0.0000626	0.00017841	0.00006718
VE	0.00044355	0.00026736	0.00009339	0.00025499	0.00009808
WMB	0.00026736	0.00043922	0.00008662	0.00026529	0.00009575
JNJ	0.00009339	0.00008662	0.00006542	0.00008531	0.00003561
DAL	0.00025499	0.00026529	0.00008531	0.00082614	0.00009229
ABT	0.00009808	0.00009575	0.00003561	0.00009229	0.00008212
TXN	0.00019303	0.00019993	0.0000659	0.00018874	0.00006992
RTN	0.00015659	0.00015964	0.00005459	0.00015158	0.00005718
OMC	0.00020473	0.00021535	0.0000688	0.00020224	0.00007352
EQR	0.00025669	0.00027386	0.00008292	0.00025549	0.00009167
IBM	0.00014317	0.00014253	0.00005185	0.00013664	0.00005285

	TXN	RTN	OMC	EQR	IBM
WMT	0.00005255	0.00004368	0.00005491	0.00006495	0.00004193
NGG	0.00010811	0.00008782	0.00011484	0.00014235	0.00008082
COH	0.00018111	0.00014494	0.0001965	0.0002412	0.00013193
JPM	0.00017597	0.00014011	0.00018724	0.00025113	0.00012142
CVX	0.00013442	0.00010845	0.00014315	0.00018114	0.00009818
VE	0.00019303	0.00015659	0.00020473	0.00025669	0.00014317
WMB	0.00019993	0.00015964	0.00021535	0.00027386	0.00014253
JNJ	0.0000659	0.00005459	0.0000688	0.00008292	0.00005185
DAL	0.00018874	0.00015158	0.00020224	0.00025549	0.00013664
ABT	0.00006992	0.00005718	0.00007352	0.00009167	0.00005285
TXN	0.00027678	0.00011589	0.00015502	0.000193	0.00010526
RTN	0.00011589	0.00016738	0.00012402	0.00015389	0.0000855
OMC	0.00015502	0.00012402	0.00024206	0.00020836	0.0001122
EQR	0.000193	0.00015389	0.00020836	0.00041694	0.00013735
IBM	0.00010526	0.0000855	0.0001122	0.00013735	0.00012591

Model II

	WMT	NGG	COH	JPM	CVX
WMT	0.00007779	0.00003948	0.00006591	0.00006677	0.00005001
NGG	0.00003948	0.00024082	0.0001332	0.00013601	0.00010125
COH	0.00006591	0.0001332	0.00043231	0.00022564	0.00016865
JPM	0.00006677	0.00013601	0.00022564	0.00037588	0.00017187
CVX	0.00005001	0.00010125	0.00016865	0.00017187	0.00017205
VE	0.00007123	0.00014497	0.00024064	0.00024743	0.00018323
WMB	0.00007244	0.00014747	0.00024475	0.00025176	0.00018638
JNJ	0.00002451	0.00004977	0.00008274	0.00008476	0.00006294
DAL	0.00006892	0.00014093	0.00023322	0.00024169	0.00017792
ABT	0.00002583	0.00005284	0.00008741	0.00009066	0.0000667
TXN	0.00005388	0.00010816	0.00018117	0.00018195	0.00013717
RTN	0.00004268	0.00008705	0.00014429	0.00014889	0.00010996
OMC	0.00005676	0.00011452	0.00019119	0.00019366	0.00014505
EQR	0.00007133	0.00014449	0.0002406	0.00024539	0.00018284
IBM	0.00003894	0.00007857	0.00013116	0.00013287	0.00009951

	VE	WMB	JNJ	DAL	ABT
WMT	0.00007123	0.00007244	0.00002451	0.00006892	0.00002583
NGG	0.00014497	0.00014747	0.00004977	0.00014093	0.00005284
COH	0.00024064	0.00024475	0.00008274	0.00023322	0.00008741
JPM	0.00024743	0.00025176	0.00008476	0.00024169	0.00009066
CVX	0.00018323	0.00018638	0.00006294	0.00017792	0.0000667
VE	0.00044286	0.00026817	0.00009032	0.00025726	0.00009649
WMB	0.00026817	0.00043869	0.00009188	0.00026181	0.0000982
JNJ	0.00009032	0.00009188	0.00006535	0.00008796	0.00003298
DAL	0.00025726	0.00026181	0.00008796	0.00082381	0.00009465
ABT	0.00009649	0.0000982	0.00003298	0.00009465	0.00008195
TXN	0.00019426	0.0001975	0.00006701	0.000187	0.00007004
RTN	0.00015855	0.00016133	0.00005428	0.00015508	0.00005818
OMC	0.00020659	0.00021009	0.00007109	0.00019988	0.0000749
EQR	0.0002616	0.0002661	0.00008985	0.00025414	0.00009527
IBM	0.00014174	0.00014415	0.00004877	0.00013716	0.0000514

	TXN	RTN	OMC	EQR	IBM
WMT	0.00005388	0.00004268	0.00005676	0.00007133	0.00003894
NGG	0.00010816	0.00008705	0.00011452	0.00014449	0.00007857
COH	0.00018117	0.00014429	0.00019119	0.0002406	0.00013116
JPM	0.00018195	0.00014889	0.00019366	0.00024539	0.00013287
CVX	0.00013717	0.00010996	0.00014505	0.00018284	0.00009951
VE	0.00019426	0.00015855	0.00020659	0.0002616	0.00014174
WMB	0.0001975	0.00016133	0.00021009	0.0002661	0.00014415
JNJ	0.00006701	0.00005428	0.00007109	0.00008985	0.00004877
DAL	0.000187	0.00015508	0.00019988	0.00025414	0.00013716
ABT	0.00007004	0.00005818	0.0000749	0.00009527	0.0000514
TXN	0.00027621	0.00011612	0.00015629	0.00019555	0.00010721
RTN	0.00011612	0.00016706	0.00012377	0.00015703	0.00008493
OMC	0.00015629	0.00012377	0.00024179	0.0002069	0.00011295
EQR	0.00019555	0.00015703	0.0002069	0.00041635	0.00014194
IBM	0.00010721	0.00008493	0.00011295	0.00014194	0.00012574

Model III

	WMT	NGG	COH	JPM	CVX
WMT	0.00007779	0.00003933	0.00006529	0.00006704	0.00004967
NGG	0.00003933	0.00024082	0.00013349	0.00013618	0.00010148
COH	0.00006529	0.00013349	0.0004323	0.00022659	0.0001691
JPM	0.00006704	0.00013618	0.00022659	0.00037589	0.00017229
CVX	0.00004967	0.00010148	0.0001691	0.00017229	0.00017205
VE	0.00007161	0.00014493	0.00024094	0.0002467	0.00018324
WMB	0.00007187	0.00014866	0.00024846	0.00025154	0.0001887
JNJ	0.0000246	0.0000497	0.00008259	0.00008465	0.00006282
DAL	0.00006925	0.00014156	0.0002359	0.00024029	0.0001793
ABT	0.00002611	0.00005289	0.00008793	0.00009001	0.00006687
TXN	0.00005335	0.00010775	0.00017903	0.00018352	0.00013618
RTN	0.00004314	0.0000869	0.00014428	0.00014811	0.00010977
OMC	0.00005634	0.00011444	0.00019042	0.00019461	0.00014479
EQR	0.00007162	0.00014392	0.00023882	0.00024547	0.00018172
IBM	0.00003868	0.00007848	0.00013054	0.0001335	0.00009927

	VE	WMB	JNJ	DAL	ABT
WMT	0.00007161	0.00007187	0.0000246	0.00006925	0.00002611
NGG	0.00014493	0.00014866	0.0000497	0.00014156	0.00005289
COH	0.00024094	0.00024846	0.00008259	0.0002359	0.00008793
JPM	0.0002467	0.00025154	0.00008465	0.00024029	0.00009001
CVX	0.00018324	0.0001887	0.00006282	0.0001793	0.00006687
VE	0.00044286	0.00026654	0.00009033	0.00025552	0.00009598
WMB	0.00026654	0.00043863	0.0000912	0.0002634	0.00009733
JNJ	0.00009033	0.0000912	0.00006535	0.00008759	0.00003295
DAL	0.00025552	0.0002634	0.00008759	0.00082382	0.00009325
ABT	0.00009598	0.00009733	0.00003295	0.00009325	0.00008195
TXN	0.00019585	0.00019767	0.00006726	0.00018988	0.00007143
RTN	0.00015819	0.00015889	0.00005435	0.00015303	0.00005769
OMC	0.00020734	0.00021135	0.00007114	0.00020193	0.00007565
EQR	0.00026237	0.0002624	0.00009017	0.0002533	0.00009567
IBM	0.00014228	0.00014474	0.00004883	0.00013844	0.00005191

	TXN	RTN	OMC	EQR	IBM
WMT	0.00005335	0.00004314	0.00005634	0.00007162	0.00003868
NGG	0.00010775	0.0000869	0.00011444	0.00014392	0.00007848
COH	0.00017903	0.00014428	0.00019042	0.00023882	0.00013054
JPM	0.00018352	0.00014811	0.00019461	0.00024547	0.0001335
CVX	0.00013618	0.00010977	0.00014479	0.00018172	0.00009927
VE	0.00019585	0.00015819	0.00020734	0.00026237	0.00014228
WMB	0.00019767	0.00015889	0.00021135	0.0002624	0.00014474
JNJ	0.00006726	0.00005435	0.00007114	0.00009017	0.00004883
DAL	0.00018988	0.00015303	0.00020193	0.0002533	0.00013844
ABT	0.00007143	0.00005769	0.00007565	0.00009567	0.00005191
TXN	0.00027623	0.00011785	0.00015424	0.00019554	0.00010587
RTN	0.00011785	0.00016707	0.00012449	0.00015819	0.00008547
OMC	0.00015424	0.00012449	0.0002418	0.00020632	0.0001122
EQR	0.00019554	0.00015819	0.00020632	0.00041634	0.00014168
IBM	0.00010587	0.00008547	0.0001122	0.00014168	0.00012574

Model IV

	WMT	NGG	COH	JPM	CVX
WMT	0.00007817	0.00004246	0.00006528	0.00005405	0.0000493
NGG	0.00004246	0.00024207	0.00013391	0.0001308	0.0001021
COH	0.00006528	0.00013391	0.00043384	0.00020866	0.00016548
JPM	0.00005405	0.0001308	0.00020866	0.00037635	0.00017466
CVX	0.0000493	0.0001021	0.00016548	0.00017466	0.0001724
VE	0.0000742	0.00014843	0.00023644	0.00024318	0.00018366
WMB	0.00006775	0.00014808	0.00024842	0.00026248	0.00018835
JNJ	0.00002993	0.00005301	0.00008064	0.00007275	0.0000624
DAL	0.00006725	0.00014175	0.00023284	0.00024253	0.00017771
ABT	0.00002846	0.00005482	0.0000845	0.00008671	0.00006681
TXN	0.00005252	0.00010801	0.00018254	0.00017643	0.00013524
RTN	0.0000437	0.00008786	0.00014429	0.0001399	0.00010808
OMC	0.00005488	0.00011479	0.00019734	0.00018751	0.00014363
EQR	0.00006494	0.00014231	0.00024179	0.00025132	0.00018148
IBM	0.00004192	0.00008079	0.00013228	0.00012153	0.00009838

	VE	WMB	JNJ	DAL	ABT
WMT	0.0000742	0.00006775	0.00002993	0.00006725	0.00002846
NGG	0.00014843	0.00014808	0.00005301	0.00014175	0.00005482
COH	0.00023644	0.00024842	0.00008064	0.00023284	0.0000845
JPM	0.00024318	0.00026248	0.00007275	0.00024253	0.00008671
CVX	0.00018366	0.00018835	0.0000624	0.00017771	0.00006681
VE	0.00044425	0.00026774	0.00009379	0.00025645	0.00009885
WMB	0.00026774	0.00043985	0.00008681	0.00026598	0.0000961
JNJ	0.00009379	0.00008681	0.00006554	0.00008606	0.00003601
DAL	0.00025645	0.00026598	0.00008606	0.00082843	0.0000937
ABT	0.00009885	0.0000961	0.00003601	0.0000937	0.0000823
TXN	0.00019134	0.00019914	0.00006504	0.00018563	0.00006828
RTN	0.00015736	0.00016	0.00005499	0.00015299	0.00005792
OMC	0.00020374	0.00021489	0.00006829	0.00020043	0.00007257
EQR	0.00025599	0.00027353	0.00008256	0.0002542	0.00009099
IBM	0.00014276	0.00014234	0.00005164	0.00013588	0.00005245

	TXN	RTN	OMC	EQR	IBM
WMT	0.00005252	0.0000437	0.00005488	0.00006494	0.00004192
NGG	0.00010801	0.00008786	0.00011479	0.00014231	0.00008079
COH	0.00018254	0.00014429	0.00019734	0.00024179	0.00013228
JPM	0.00017643	0.0001399	0.00018751	0.00025132	0.00012153
CVX	0.00013524	0.00010808	0.00014363	0.00018148	0.00009838
VE	0.00019134	0.00015736	0.00020374	0.00025599	0.00014276
WMB	0.00019914	0.00016	0.00021489	0.00027353	0.00014234
JNJ	0.00006504	0.00005499	0.00006829	0.00008256	0.00005164
DAL	0.00018563	0.00015299	0.00020043	0.0002542	0.00013588
ABT	0.00006828	0.00005792	0.00007257	0.00009099	0.00005245
TXN	0.00027726	0.00011425	0.00015712	0.00019449	0.00010613
RTN	0.00011425	0.00016766	0.00012306	0.00015321	0.0000851
OMC	0.00015712	0.00012306	0.00024234	0.00020924	0.00011271
EQR	0.00019449	0.00015321	0.00020924	0.00041754	0.00013771
IBM	0.00010613	0.0000851	0.00011271	0.00013771	0.0001261

Model V

	WMT	NGG	COH	JPM	CVX
WMT	0.00007817	0.0000424	0.00006518	0.00005408	0.00004919
NGG	0.0000424	0.00024207	0.00013407	0.00013078	0.00010226
COH	0.00006518	0.00013407	0.00043384	0.00020841	0.00016541
JPM	0.00005408	0.00013078	0.00020841	0.00037635	0.00017448
CVX	0.00004919	0.00010226	0.00016541	0.00017448	0.0001724
VE	0.00007431	0.00014825	0.00023686	0.00024347	0.00018377
WMB	0.00006722	0.00014865	0.00024984	0.00026225	0.00018974
JNJ	0.00002995	0.00005295	0.00008092	0.00007288	0.00006253
DAL	0.00006706	0.00014184	0.0002344	0.00024282	0.00017878
ABT	0.00002846	0.00005476	0.00008511	0.00008693	0.00006713
TXN	0.00005274	0.0001079	0.00018073	0.00017609	0.00013399
RTN	0.00004387	0.0000876	0.00014455	0.00014023	0.00010802
OMC	0.00005498	0.00011476	0.00019634	0.00018729	0.00014297
EQR	0.0000654	0.00014184	0.00024025	0.00025142	0.00018009
IBM	0.00004196	0.00008079	0.00013187	0.00012144	0.00009811

	VE	WMB	JNJ	DAL	ABT
WMT	0.00007431	0.00006722	0.00002995	0.00006706	0.00002846
NGG	0.00014825	0.00014865	0.00005295	0.00014184	0.00005476
COH	0.00023686	0.00024984	0.00008092	0.0002344	0.00008511
JPM	0.00024347	0.00026225	0.00007288	0.00024282	0.00008693
CVX	0.00018377	0.00018974	0.00006253	0.00017878	0.00006713
VE	0.00044426	0.00026616	0.00009345	0.00025463	0.00009812
WMB	0.00026616	0.00043976	0.00008634	0.00026687	0.00009555
JNJ	0.00009345	0.00008634	0.00006554	0.00008523	0.00003562
DAL	0.00025463	0.00026687	0.00008523	0.00082847	0.00009223
ABT	0.00009812	0.00009555	0.00003562	0.00009223	0.00008231
TXN	0.00019346	0.00019809	0.000066	0.00018819	0.00006999
RTN	0.00015702	0.00015775	0.0000547	0.00015101	0.00005724
OMC	0.00020491	0.00021459	0.00006884	0.00020202	0.00007355
EQR	0.00025773	0.00026932	0.00008317	0.00025413	0.00009184
IBM	0.00014323	0.00014225	0.00005186	0.00013655	0.00005286

	TXN	RTN	OMC	EQR	IBM
WMT	0.00005274	0.00004387	0.00005498	0.0000654	0.00004196
NGG	0.0001079	0.0000876	0.00011476	0.00014184	0.00008079
COH	0.00018073	0.00014455	0.00019634	0.00024025	0.00013187
JPM	0.00017609	0.00014023	0.00018729	0.00025142	0.00012144
CVX	0.00013399	0.00010802	0.00014297	0.00018009	0.00009811
VE	0.00019346	0.00015702	0.00020491	0.00025773	0.00014323
WMB	0.00019809	0.00015775	0.00021459	0.00026932	0.00014225
JNJ	0.000066	0.0000547	0.00006884	0.00008317	0.00005186
DAL	0.00018819	0.00015101	0.00020202	0.00025413	0.00013655
ABT	0.00006999	0.00005724	0.00007355	0.00009184	0.00005286
TXN	0.0002773	0.00011655	0.00015528	0.00019459	0.00010535
RTN	0.00011655	0.00016766	0.00012429	0.00015552	0.0000856
OMC	0.00015528	0.00012429	0.00024236	0.00020902	0.00011224
EQR	0.00019459	0.00015552	0.00020902	0.00041749	0.00013758
IBM	0.00010535	0.0000856	0.00011224	0.00013758	0.0001261

Model VI

	WMT	NGG	COH	JPM	CVX
WMT	0.00007802	0.00003936	0.0000656	0.0000667	0.00004979
NGG	0.00003936	0.00024146	0.00013358	0.00013609	0.00010151
COH	0.0000656	0.00013358	0.00043314	0.00022586	0.00016935
JPM	0.0000667	0.00013609	0.00022586	0.00037646	0.00017202
CVX	0.00004979	0.00010151	0.00016935	0.00017202	0.00017222
VE	0.00007133	0.00014486	0.00024033	0.00024737	0.00018301
WMB	0.00007152	0.00014856	0.00024767	0.00025239	0.0001884
JNJ	0.00002458	0.0000497	0.00008253	0.00008471	0.0000628
DAL	0.00006856	0.00014136	0.00023439	0.00024194	0.00017873
ABT	0.00002585	0.00005281	0.00008735	0.00009064	0.00006665
TXN	0.00005406	0.00010795	0.00018059	0.00018182	0.00013677
RTN	0.00004287	0.00008682	0.00014368	0.00014876	0.00010954
OMC	0.00005674	0.00011455	0.00019128	0.00019368	0.00014511
EQR	0.00007178	0.00014396	0.00023917	0.00024509	0.00018185
IBM	0.00003895	0.00007855	0.00013113	0.00013287	0.00009949

	VE	WMB	JNJ	DAL	ABT
WMT	0.00007133	0.00007152	0.00002458	0.00006856	0.00002585
NGG	0.00014486	0.00014856	0.0000497	0.00014136	0.00005281
COH	0.00024033	0.00024767	0.00008253	0.00023439	0.00008735
JPM	0.00024737	0.00025239	0.00008471	0.00024194	0.00009064
CVX	0.00018301	0.0001884	0.0000628	0.00017873	0.00006665
VE	0.00044358	0.00026726	0.00009038	0.0002569	0.00009651
WMB	0.00026726	0.00043925	0.00009127	0.00026517	0.00009802
JNJ	0.00009038	0.00009127	0.00006549	0.00008772	0.000033
DAL	0.0002569	0.00026517	0.00008772	0.0008261	0.00009458
ABT	0.00009651	0.00009802	0.000033	0.00009458	0.00008214
TXN	0.00019443	0.00019585	0.00006713	0.00018635	0.00007007
RTN	0.00015874	0.00015959	0.0000544	0.00015438	0.00005821
OMC	0.00020656	0.00021035	0.00007107	0.00019999	0.0000749
EQR	0.00026205	0.00026198	0.00009014	0.00025251	0.00009536
IBM	0.00014175	0.00014406	0.00004878	0.00013712	0.0000514

	TXN	RTN	OMC	EQR	IBM
WMT	0.00005406	0.00004287	0.00005674	0.00007178	0.00003895
NGG	0.00010795	0.00008682	0.00011455	0.00014396	0.00007855
COH	0.00018059	0.00014368	0.00019128	0.00023917	0.00013113
JPM	0.00018182	0.00014876	0.00019368	0.00024509	0.00013287
CVX	0.00013677	0.00010954	0.00014511	0.00018185	0.00009949
VE	0.00019443	0.00015874	0.00020656	0.00026205	0.00014175
WMB	0.00019585	0.00015959	0.00021035	0.00026198	0.00014406
JNJ	0.00006713	0.0000544	0.00007107	0.00009014	0.00004878
DAL	0.00018635	0.00015438	0.00019999	0.00025251	0.00013712
ABT	0.00007007	0.00005821	0.0000749	0.00009536	0.0000514
TXN	0.00027671	0.00011646	0.00015624	0.00019636	0.00010723
RTN	0.00011646	0.00016735	0.00012372	0.00015788	0.00008495
OMC	0.00015624	0.00012372	0.0002421	0.00020677	0.00011295
EQR	0.00019636	0.00015788	0.00020677	0.00041695	0.00014199
IBM	0.00010723	0.00008495	0.00011295	0.00014199	0.00012593

Model VII

	WMT	NGG	COH	JPM	CVX
WMT	0.00007839	0.0000424	0.00006518	0.00005408	0.00004919
NGG	0.0000424	0.00024271	0.00013402	0.00013076	0.00010223
COH	0.00006518	0.00013402	0.00043465	0.00020859	0.0001657
JPM	0.00005408	0.00013076	0.00020859	0.00037676	0.00017458
CVX	0.00004919	0.00010223	0.0001657	0.00017458	0.00017257
VE	0.00007431	0.0001483	0.00023623	0.00024325	0.00018342
WMB	0.00006722	0.00014869	0.00024944	0.00026211	0.00018952
JNJ	0.00002995	0.00005298	0.00008059	0.00007277	0.00006235
DAL	0.00006707	0.00014196	0.00023318	0.0002424	0.0001781
ABT	0.00002847	0.00005481	0.00008449	0.00008671	0.00006679
TXN	0.00005272	0.00010777	0.00018215	0.00017658	0.00013478
RTN	0.00004387	0.00008766	0.00014395	0.00014002	0.00010769
OMC	0.00005497	0.00011468	0.00019716	0.00018757	0.00014343
EQR	0.00006539	0.00014178	0.00024091	0.00025164	0.00018045
IBM	0.00004195	0.00008076	0.00013221	0.00012156	0.0000983

	VE	WMB	JNJ	DAL	ABT
WMT	0.00007431	0.00006722	0.00002995	0.00006707	0.00002847
NGG	0.0001483	0.00014869	0.00005298	0.00014196	0.00005481
COH	0.00023623	0.00024944	0.00008059	0.00023318	0.00008449
JPM	0.00024325	0.00026211	0.00007277	0.0002424	0.00008671
CVX	0.00018342	0.00018952	0.00006235	0.0001781	0.00006679
VE	0.00044497	0.00026663	0.00009384	0.00025608	0.00009886
WMB	0.00026663	0.00044037	0.00008658	0.00026778	0.00009602
JNJ	0.00009384	0.00008658	0.00006565	0.00008598	0.00003601
DAL	0.00025608	0.00026778	0.00008598	0.00083076	0.00009367
ABT	0.00009886	0.00009602	0.00003601	0.00009367	0.00008248
TXN	0.00019177	0.00019704	0.00006513	0.00018493	0.00006832
RTN	0.00015773	0.00015819	0.00005506	0.00015239	0.00005795
OMC	0.00020393	0.00021398	0.00006833	0.00020012	0.00007258
EQR	0.00025695	0.00026884	0.00008276	0.00025263	0.00009107
IBM	0.00014283	0.000142	0.00005165	0.00013577	0.00005245

	TXN	RTN	OMC	EQR	IBM
WMT	0.00005272	0.00004387	0.00005497	0.00006539	0.00004195
NGG	0.00010777	0.00008766	0.00011468	0.00014178	0.00008076
COH	0.00018215	0.00014395	0.00019716	0.00024091	0.00013221
JPM	0.00017658	0.00014002	0.00018757	0.00025164	0.00012156
CVX	0.00013478	0.00010769	0.00014343	0.00018045	0.0000983
VE	0.00019177	0.00015773	0.00020393	0.00025695	0.00014283
WMB	0.00019704	0.00015819	0.00021398	0.00026884	0.000142
JNJ	0.00006513	0.00005506	0.00006833	0.00008276	0.00005165
DAL	0.00018493	0.00015239	0.00020012	0.00025263	0.00013577
ABT	0.00006832	0.00005795	0.00007258	0.00009107	0.00005245
TXN	0.00027777	0.00011495	0.00015748	0.00019633	0.00010626
RTN	0.00011495	0.00016795	0.00012337	0.00015478	0.00008521
OMC	0.00015748	0.00012337	0.00024264	0.00021003	0.00011277
EQR	0.00019633	0.00015478	0.00021003	0.00041808	0.000138
IBM	0.00010626	0.00008521	0.00011277	0.000138	0.00012629

Actual Return Covariance Matrix

	WMT	NGG	COH	JPM	CVX
WMT	0.00010965	0.00006419	0.00011545	0.00014078	0.00009158
NGG	0.00006419	0.00018942	0.00013809	0.00016024	0.00012417
COH	0.00011545	0.00013809	0.00062198	0.00035191	0.0002543
JPM	0.00014078	0.00016024	0.00035191	0.00058334	0.00028152
CVX	0.00009158	0.00012417	0.0002543	0.00028152	0.00028079
VE	0.00018225	0.00027507	0.00049628	0.00060088	0.00040766
WMB	0.00013037	0.00016739	0.00039645	0.00041114	0.00033276
JNJ	0.00006212	0.00007654	0.00013841	0.0001669	0.00012468
DAL	0.00015318	0.0001545	0.00036079	0.00040613	0.00018782
ABT	0.00006586	0.0000823	0.0001112	0.00014471	0.00011299
TXN	0.00007843	0.00010738	0.00027057	0.00023436	0.00019355
RTN	0.00009486	0.00010781	0.00020617	0.00024385	0.00015884
OMC	0.00010113	0.00012938	0.00032742	0.00031152	0.00021514
EQR	0.00010966	0.00012419	0.00029951	0.00032846	0.00021606
IBM	0.00007041	0.00010118	0.00021853	0.00019432	0.00014056

	VE	WMB	JNJ	DAL	ABT
WMT	0.00018225	0.00013037	0.00006212	0.00015318	0.00006586
NGG	0.00027507	0.00016739	0.00007654	0.0001545	0.0000823
COH	0.00049628	0.00039645	0.00013841	0.00036079	0.0001112
JPM	0.00060088	0.00041114	0.0001669	0.00040613	0.00014471
CVX	0.00040766	0.00033276	0.00012468	0.00018782	0.00011299
VE	0.00117312	0.0005978	0.00023846	0.00045399	0.00022699
WMB	0.0005978	0.00064611	0.00015909	0.00033359	0.0001413
JNJ	0.00023846	0.00015909	0.00011803	0.00014234	0.00007593
DAL	0.00045399	0.00033359	0.00014234	0.00101496	0.0001377
ABT	0.00022699	0.0001413	0.00007593	0.0001377	0.00012515
TXN	0.00034957	0.00026803	0.00009941	0.00025526	0.00008621
RTN	0.00034246	0.00022635	0.00009944	0.0002314	0.00009957
OMC	0.00041511	0.00031932	0.00011759	0.00028672	0.00011312
EQR	0.00042024	0.00031602	0.0001184	0.00029402	0.00010283
IBM	0.00029942	0.00020511	0.00008683	0.00018072	0.00009096

	TXN	RTN	OMC	EQR	IBM
WMT	0.00007843	0.00009486	0.00010113	0.00010966	0.00007041
NGG	0.00010738	0.00010781	0.00012938	0.00012419	0.00010118
COH	0.00027057	0.00020617	0.00032742	0.00029951	0.00021853
JPM	0.00023436	0.00024385	0.00031152	0.00032846	0.00019432
CVX	0.00019355	0.00015884	0.00021514	0.00021606	0.00014056
VE	0.00034957	0.00034246	0.00041511	0.00042024	0.00029942
WMB	0.00026803	0.00022635	0.00031932	0.00031602	0.00020511
JNJ	0.00009941	0.00009944	0.00011759	0.0001184	0.00008683
DAL	0.00025526	0.0002314	0.00028672	0.00029402	0.00018072
ABT	0.00008621	0.00009957	0.00011312	0.00010283	0.00009096
TXN	0.00029474	0.00015472	0.00022297	0.00018082	0.00013962
RTN	0.00015472	0.00022258	0.00018497	0.0001839	0.00012815
OMC	0.00022297	0.00018497	0.00032959	0.00023019	0.00017464
EQR	0.00018082	0.0001839	0.00023019	0.00035572	0.00014017
IBM	0.00013962	0.00012815	0.00017464	0.00014017	0.0001948

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